K24U 4021

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admission)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01MAT-CH : Mathematics for Chemistry – I

Time : 3 Hours   Max. Marks : 40

SECTION – A

Questions **1-5**, answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. If y = (ax + b) (cx + d), show that $2y_1y_2 = y_2^2$.
2. State Lagrange's mean value theorem.
3. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$.
4. Does the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ is an elementary matrix ? Justify your answer.
5. Show that the matrices $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ are equivalent matrices.

SECTION – B

Questions **6-15**, answer **any seven** questions. **Each** question carries **two** marks. **(7×2=14)**

6. Show that $D^n (\sin(ax + b)) = a^n \sin(ax + b + n\pi/2)$.
7. If $x = a(\cos t + t \sin t), y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.
8. Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos 4x}$.

P.T.O.

9. Prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .$

10. Find the normal form of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 1 \\ 2 & 0 & 0 \end{pmatrix}$.

11. Determine the value of p such that the rank of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 2 & p & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is 1.

12. If A is orthogonal, show that $|A| = \pm 1$.

13. Convert the curve $y = 3e^{2x}$ in to a straight line.

14. Write the normal equations corresponding to the straight line y = ax + b.

15. Explain briefly on the method of least squares to fit the parabola $y = a + bx + cx^2$.

SECTION – C

Questions **16-22**, answer **any four** questions. **Each** question carries **three** marks.

(4×3=12)

16. Given that $y = e^{a \sin^{-1} x}$. Show that $(1 - x^2) y_2 - xy_1 - a^2y = 0$.

17. If $y = x^n \log x$. Prove that $y_{n+1} = n!/x$.

18. Expand $\log_e x$ in terms of x – 1 and evaluate $\log_e 1.1$ correct to four decimal places.

19. Verify the result of Cauchy's mean value theorem for the functions sin x and cos x in the interval [a, b].

20. Solve the system of equations : 2x + y + z = 2, x – y + z = 0, –x – y + 3z = 2 using the Cramer's rule.

21. Reduce the matrix A to its normal form where $A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ and hence find the rank of A.

22. R is the resistance to maintain a train at speed V, find a law of the type $R = a + bV^2$ connecting R and V, using the following data.

V	R
10	8
30	15
40	21
50	30

SECTION – D

Questions **23-26**, answer **any two** questions. **Each** question carries **five** marks.

(2×5=10)

23. If $y = \tan^{-1} x$, prove that $(1 + x^2)\, y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

24. Show that $\lim\limits_{x \to 0} \dfrac{x^x - x}{x - 1 - \log x} = 2$.

25. Show that the equations $5x + 3y + 7z - 4 = 0$, $x + 26y + 2z - 9 = 0$, $7x + 2y + 10z - 5 = 0$ are consistent and solve.

26. Fit a second degree parabola to the following data.

x	y
0	1
1	1.8
2	1.3
3	2.5
4	6.3